

If $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\phi)$ are independent random variables, then $X+Y \sim \text{Po}(\lambda+\phi)$

If $X \sim B(n, p)$ and n is large and p is small, then $Y \sim \text{Po}(np)$.

Read pages 134 & 135 (Three different scenarios for Poisson hypothesis tests!)

① Has rate changed/increased/decreased?

$H_0: \lambda = ?$

$H_1: \lambda \neq ?$ or $\lambda < ?$ or $\lambda > ?$

Significance level = 10% = 0.1 (or different significance)

Find p value $P(X \geq \text{new value}) = p$ or $P(X \leq \text{new value}) = p$
half this if test is two-tailed which we are testing for

If $p < \text{significance level} \Rightarrow \text{result significant} \Rightarrow \text{reject } H_0$

If $p > \text{significance level} \Rightarrow \text{not sig} \Rightarrow \text{do not reject } H_0$

② Find critical region.

This involves finding upper/lower tails as close to the significance level as possible (probably won't be exact)

eg $P(X \leq 1) = 0.0072$ and $P(X \geq 15) = 0.0057$

Question may ask for actual significance level.

This is $P(X \leq 1) + P(X \geq 15) = 0.0072 + 0.0057 = 0.013$

Question may then ask about a particular value, say 6; Student can say this is outside critical region and then conclude: (there is insufficient evidence...)

③ A binomial scenario with large n and small p . Convert this to Poisson and then use either of the two above techniques

Geometric Hypothesis Tests similar to points ① and ② above

for Poisson Hypothesis Tests

In order to find $P(X \leq ?)$ or $P(X \geq ?)$, you could use formulae:

$$S_n = \frac{q(1-r^n)}{1-r} \quad \text{or} \quad S_\infty = \frac{q}{1-r}$$

Chi-squared tests checks whether a model is suitable or not (it could be a Binomial model or Poisson etc) Use expected and observed values.

Usually, degrees of freedom = $n-1$

If λ is estimated, then deg of freedom = $n-2$

If Test Statistic < Critical Value \Rightarrow not significant \Rightarrow do not reject H_0

If " " " > " " \Rightarrow result significant \Rightarrow reject H_0

Finally state concluding sentences

For two variables, Chi-squared works as follows:

$$\text{Expected Values } E_i = \frac{(\text{row total}) \times (\text{column total})}{\text{overall total } N}$$

$$\text{No. of degrees of freedom} = \underset{\substack{\uparrow \\ \text{no of rows}}}{(r-1)} \times \underset{\substack{\uparrow \\ \text{no of columns}}}{(c-1)}$$

Quality of Tests: read pages 142 & 143 for a good example.

Type I Error - rejecting H_0 when it is actually true

Type II Error - not rejecting H_0 " " " false.

Size of a test is probability of Type I error (or probability of being in critical region given H_0 is correct)

Power of a test is probability of correctly rejection H_0 .

$$\text{Power of Test} = 1 - P(\text{Type II Error})$$