Ref: P61177A
3 June 2019

① $f(z)=z^4+az^3+bz^2+cz+d$

where a, b and c are real constants.

Given that $-1+2i$ and $3-i$ are two roots of the equation $f(z)=0$

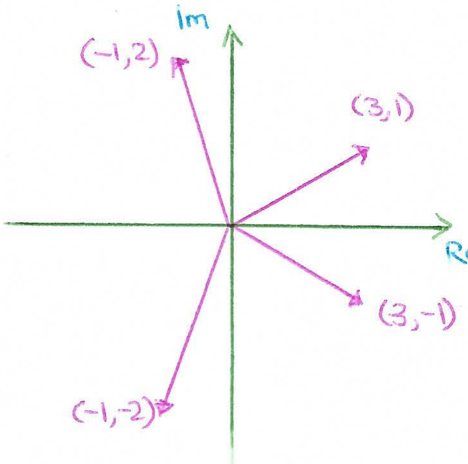
ⓐ Show all the roots of $f(z)=0$ on a single Argand diagram (4)

ⓑ find the values a, b, c and d. (5)

Solution 1a

$z_1=-1+2i \Rightarrow z_2=-1-2i$

$z_3=3-i \Rightarrow z_4=3+i$



Solution 1b

$f(z)=(z-z_1)(z-z_2)(z-z_3)(z-z_4)$

$=(z-(-1+2i))(z-(-1-2i))(z-(3-i))(z-(3+i))$

$=(z^2-z(-1-2i)-z(-1+2i)+(-1+2i)(-1-2i))(z^2-z(3+i)-z(3-i)+(9+1))$

$=(z^2+z+z2i+z-z2i+1+4)(z^2-3z-zi-3z+zi+10)$

$=(z^2+2z+5)(z^2-6z+10)$

$=z^4-6z^3+10z^2+2z^3-12z^2+20z+5z^2-30z+50$

$=z^4-4z^3+3z^2-10z+50$

$\Rightarrow a=-4,\ b=3,\ c=-10,\ d=50$

② Show that

$$\int_0^\infty \frac{8x-12}{(2x^2+3)(x+1)}\,dx = \ln k$$

where k is a rational number to be found. (7)

Solution 2

Convert fraction to partial fractions:

Let $\dfrac{8x-12}{(2x^2+3)(x+1)} \equiv \dfrac{Ax+B}{2x^2+3} + \dfrac{C}{x+1}$

$\Rightarrow 8x-12 \equiv (Ax+B)(x+1) + C(2x^2+3)$

$x=0 \Rightarrow -12 = B + 3C$ (*)

$x=-1 \Rightarrow -20 = 5C \Rightarrow C = -4$

Substitute $C=-4$ in (*)

$-12 = B - 12 \Rightarrow B = 0$

$x=1 \Rightarrow 8-12 = (A+\underbrace{0}_{B=0})(1+1) + (\underbrace{-4}_{C=-4})(2+3)$

$\Rightarrow -4 = 2A - 20$

$\Rightarrow 2A = 16 \Rightarrow A = 8$

Hence $\displaystyle\int_0^\infty \frac{8x-12}{(2x^2+3)(x+1)}\,dx = \int_0^\infty \frac{8x}{2x^2+3} - \frac{4}{x+1}\,dx$

$\displaystyle = \int_0^\infty 2\cdot\frac{4x}{2x^2+3} - 4\cdot\frac{1}{x+1}\,dx$

$= \left[2\ln(2x^2+3) - 4\ln(x+1)\right]_0^\infty$

$= \left[2\ln(2x^2+3) - 2\times 2\ln(x+1)\right]_0^\infty$

$= \left[2\ln(2x^2+3) - 2\ln\left((x+1)^2\right)\right]_0^\infty$

Solution 2 continued

$$\int_0^\infty \frac{8x-12}{(2x^2+3)(x+1)}\,dx$$

$$= 2\left[\ln(2x^2+3) - \ln\left((x+1)^2\right)\right]_0^\infty$$

$$= 2\left[\ln\left(\frac{2x^2+3}{(x+1)^2}\right)\right]_0^\infty$$

$$= 2\left[\ln\left(\frac{2x^2+3}{x^2+2x+1}\right)\right]_0^\infty$$

$$= 2\lim_{t\to\infty}\left(\ln\left(\frac{2t^2+3}{t^2+2t+1}\right) - \ln\left(\frac{2\times 0^2+3}{0^2+2\times 0+1}\right)\right)$$

$$= 2\lim_{t\to\infty}\left(\ln\left(\frac{2+3/t^2}{1+2/t+1/t^2}\right) - \ln(3)\right)$$

$$= 2\left(\ln\left(\frac{2}{1}\right) - \ln 3\right)$$

$$= 2\ln\left(\frac{2}{3}\right) = \ln\left(\left(\frac{2}{3}\right)^2\right) = \ln\left(\frac{4}{9}\right)$$

(3)

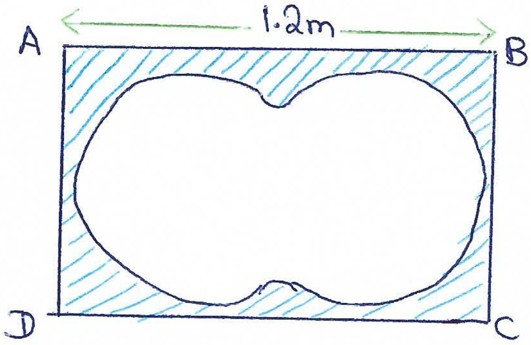


Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle ABCD. The length of the table, AB is 1.2m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leq \theta < 2\pi$$

where a is a constant.

(a) Show that $a = 0.2$ (2)

Hence, given that $AD = 60\text{cm}$,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures. (8)

Solution 3a

Values of $\cos\theta$ go from -1 to $+1$.

$$2(0.4 + a) = 1.2 \quad \Rightarrow \quad a = 0.2$$

Solution 3b

Recall: Area $= \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$

Area enclosed by curve $= \frac{1}{2}\int_0^{2\pi} (0.4 + 0.2\cos(2\theta))^2 \, d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(\boxed{0.4^2} + \boxed{2 \times 0.2 \times 0.4\cos(2\theta)} + \boxed{0.2^2 \cos^2(2\theta)} \right) d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(\boxed{0.16} + \boxed{0.16\cos 2\theta} + \boxed{0.04\cos^2(2\theta)} \right) d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(\boxed{0.16} + \boxed{0.16\cos 2\theta} + \boxed{0.04\left(\frac{\cos 4\theta + 1}{2}\right)} \right) d\theta$

$= \frac{1}{2}\int_0^{2\pi} \left(\boxed{0.16} + \boxed{0.16\cos 2\theta} + \boxed{0.02\cos 4\theta + 0.02} \right) d\theta$

0.16+0.02

$= \frac{1}{2}\left[\boxed{0.18\theta} + \boxed{0.16\frac{\sin 2\theta}{2}} + \boxed{0.02\frac{\sin 4\theta}{4}} \right]_0^{2\pi}$

$= \frac{1}{2}\left[\boxed{0.18\theta} + \boxed{0.08\sin 2\theta} + \boxed{0.005\sin 4\theta} \right]_0^{2\pi}$

$= \left[\boxed{0.09\theta} + \boxed{0.04\sin 2\theta} + \boxed{0.0025\sin 4\theta} \right]_0^{2\pi}$

$= (0.09 \times 2\pi + 0.04\sin 4\pi + 0.0025\sin 8\pi) - (0 + 0.04\sin 0 - 0.0025\sin 0)$

$= \underline{0.18\pi} \; (= 0.5654\cdots) \; m^2$

Area of rectangle $= 1.2m \times 0.6m = \underline{0.72m^2}$

Area of wood $=$ Area of rectangle $-$ Area enclosed by curve

$= \underline{0.72m^2} - \underline{0.18\pi \, m^2}$

$= 0.155m^2$

④ Prove that, for $n \in \mathbb{Z}, n \geqslant 0$

$$\sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where a, b and c are integers to be found.

Solution 4

Partial fractions conversions will hopefully result in cancellations

$$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1} + \frac{B}{r+2} + \frac{C}{r+3}$$

$$\Rightarrow \quad 1 \equiv A(r+2)(r+3) + B(r+1)(r+3) + C(r+1)(r+2)$$

$\Rightarrow$ when $r=-1$ we have $1 = A(1)(2) \Rightarrow A = \frac{1}{2}$

when $r=-2$ we have $1 = B(-1)(1) \Rightarrow B = -1$

when $r=-3$ we have $1 = C(-2)(-1) \Rightarrow C = \frac{1}{2}$

$$\text{So} \quad \sum_{r=0}^{n} \frac{1}{(r+1)(r+2)(r+3)} = \sum_{r=0}^{n} \frac{1}{2(r+1)} - \frac{1}{(r+2)} + \frac{1}{2(r+3)}$$

$$= \frac{1}{2}\sum_{r=0}^{n}\left(\frac{1}{r+1} - \frac{2}{r+2} + \frac{1}{r+3}\right)$$

$$= \frac{1}{2}\Bigg(\underbrace{\left(\frac{1}{1} - \frac{2}{2} + \frac{1}{3}\right)}_{r=0} + \underbrace{\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right)}_{r=1} + \underbrace{\left(\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right)}_{r=2} + \underbrace{\left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right)}_{r=3}$$

$$+ \dots + \underbrace{\left(\frac{1}{n-1} - \frac{2}{n} + \frac{1}{n+1}\right)}_{r=n-2} + \underbrace{\left(\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right)}_{r=n-1} + \underbrace{\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)}_{r=n}$$

Look at the pattern of cancellations.

$$= \frac{1}{2}\left(1 - 1 + \frac{1}{2} + \frac{1}{n+2} - \frac{2}{n+2} + \frac{1}{n+3}\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{n+2} + \frac{1}{n+3}\right)$$

$$= \frac{1}{4}\left(\frac{(n+2)(n+3) - 2(n+3) + 2(n+2)}{2(n+2)(n+3)}\right) = \frac{n^2+5n+6-2n-6+2n+4}{4(n+2)(n+3)}$$

$$= \frac{n^2+5n+4}{4(n+2)(n+3)} = \frac{(n+1)(n+4)}{4(n+2)(n+3)} \quad \Rightarrow a=1,\ b=4 \text{ and } c=4$$

(reference to question)

(5) A tank at a chemical plant has a capacity of 250 litres. The tank initially contains 100 litres of pure water.
Salt water enters the tank at a rate of 3 litres every minute. Each litre of salt water entering the tank contains 1 gram of salt.
It is assumed that the salt water mixes instantly with the contents of the tank upon entry.
At the instant when the salt water begins to enter the tank, a valve is opened at the bottom of the tank and the solution in the tank flows out at a rate of 2 litres per minute.
Given that there are S grams of salt in the tank after t minutes,

(a) show that the situation can be modelled by the differential equation

$$\frac{dS}{dt} = 3 - \frac{2S}{100+t}$$ (4)

(b) Hence find the number of grams of salt in the tank after 10 minutes. (5)

When the concentration of salt in the tank reaches 0.9 grams per litre, the valve at the bottom of the tank must be closed.

(c) Find, to the nearest minute, when the valve would need to be closed. (3)

(d) Evaluate the model. (1)

Solution 5a

The tank initially contains 100L.
3L are entering every minute.
2L are leaving every minute.
1L increase in volume each minute so the tank contains 100+t litres after t minutes.

Salt enters the tank at 3×1g per min

S grams of salt in tank
So concentration of salt is $\frac{S}{100+t}$ g/L

$$\frac{dS}{dt} = 3 - 2 \times \boxed{\frac{S}{100+t}}$$

2 L leave the tank every min

Solution 5b

$$\frac{dS}{dt} + \frac{2S}{100+t} = 3 \qquad (*)$$

$$I = e^{\int \frac{2}{100+t}dt} = e^{2\int \frac{1}{100+t}dt} = e^{2\ln(100+t)} = e^{\ln(100+t)^2} = (100+t)^2$$

multiplying eqn by $(100+t)^2$

$$(*) \Rightarrow (100+t)^2 \frac{dS}{dt} + 2S(100+t) = 3(100+t)^2$$

$$\Rightarrow S(100+t)^2 = 3\int (100+t)^2 dt$$

$$\Rightarrow S(100+t)^2 = 3\frac{(100+t)^3}{3} + C$$

$$\Rightarrow S(100+t)^2 = (100+t)^3 + C$$

When $t=0$, $S=0 \Rightarrow 0 = 10^6 + C \Rightarrow C = -10^6$

So $S(100+t)^2 = (100+t)^3 - 10^6 \qquad (**)$

When $t=10$, $S = \frac{(100+10)^3 - 10^6}{(100+10)^2} = 110 - \frac{10^6}{110^2} = 27g$

Solution 5c

This part is giving "concentration" of salt (not quantity of salt).

Concentration of salt = 0.9 grams

$$\Rightarrow \boxed{\frac{S}{100+t} = 0.9}$$

(**) from part b $\Rightarrow S(100+t)^2 = (100+t)^3 - 10^6$

$\div$ by $(100+t)^3$

$$\Rightarrow \boxed{\frac{S}{100+t}} = 1 - \frac{10^6}{(100+t)^3}$$

this will give us $\frac{S}{100+t}$ in our equation

$$\Rightarrow \boxed{0.9} = 1 - \frac{10^6}{(100+t)^3}$$

rearranging and tidying up

$$\Rightarrow 0.1 = \frac{10^6}{(100+t)^3}$$

$$\Rightarrow 10^6 = (100+t)^3 \times 0.1$$

$\times 10$

$$\Rightarrow 10^7 = (100+t)^3$$

cube root

$$\Rightarrow 10^{7/3} = 100+t$$

$$\Rightarrow t = 10^{7/3} - 100$$

$$\Rightarrow t \approx 115 \text{ minutes}$$

Solution 5d

Can say:- it is unlikely that mixing is instantaneous

- model is valid when tank is not full
- when valve closed, model not valid
- unlikely concentration of salt water entering tank remains exactly the same.

FM1
Paper 1: CPM1

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⑥ Prove by induction that for all positive integers n
$f(n) = 3^{2n+4} - 2^{2n}$ is divisible by 5 (6)

Solution 6

When $n=1$, $f(1) = 3^{2(1)+4} - 2^{2(1)} = 3^6 - 2^2 = 725$
$\therefore$ statement true when $n=1$.

Assume statement true when $n=k$

So $f(k) = 3^{2k+4} - 2^{2k} = 5p$ for some $p \in \mathbb{Z}$

When $n=k+1$, we have

$$f(k+1) = 3^{2(k+1)+4} - 2^{2(k+1)}$$

$$= 3^{2k+4} \cdot 3^2 - 2^{2k} \cdot 2^2$$

Because this is 4, we split the multiple of 3^{2k+4} as 4+5.

$9 = 4+5$

$$= 3^{2k+4} \times 4 + 3^{2k+4} \times 5 - 2^{2k} \times 4$$

$$= (3^{2k+4} - 2^{2k}) \cdot 4 + 5 \times 3^{2k+4}$$

$$= 5p \times 4 + 5 \times 3^{2k+4}$$ by inductive step

$$= 5(4p + 3^{2k+4})$$

$\therefore$ Statement true when $n=k+1$.

If statement is true when $n=k$ then it is true for $n=k+1$ and as it is true for $n=1$, the statement is true for all positive integers n.

⑦ The line l_1 has equation $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$

The line l_2 has equation $\underline{r} = i + 3k + t(i - j + 2k)$

where t is a scalar parameter.

ⓐ Show that l_1 and l_2 lie in the same plane. (3)

ⓑ Write down a vector equation for the plane containing l_1 and l_2 (1)

ⓒ Find, to the nearest degree, the acute angle between l_1 and l_2 (3)

Solution 7a

l_1 has equation $\frac{x-\boxed{1}}{\boxed{2}} = \frac{y+\boxed{1}}{\boxed{-1}} = \frac{z-\boxed{4}}{\boxed{3}}$

$\Rightarrow$ l_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}} + p\boxed{\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}}$ (*)

l_2 has equation $\underline{r} = \boxed{i + 3k} + t\boxed{(i - j + 2k)}$

$\Rightarrow$ l_2 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boxed{\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}} + t\boxed{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}$ (**)

If l_1 and l_2 lie in the same plane, then they meet

ie Eqn (*) = Eqn (**)

$\Leftrightarrow$ $\boxed{\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + p\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$\Leftrightarrow$ $\boxed{1 + 2p = 1 + t}$
$-1 - p = -t$
$4 + 3p = 3 + 2t$

$\Leftrightarrow$ $2p = t$ ①
$p = t - 1$ ②
$3p = 2t - 1$ ③

Solving gives
$p = 1$ and $t = 2$
So lines meet at

$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$

$\therefore$ l_1 and l_2 lie in the same plane.

Solution 7b

A vector equation for the plane containing l_1 and l_2:

direction vector for l_1

direction vector for l_2

$$\underline{r} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

vector where lines meet

Solution 7c

Recall: $\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}|\cos\theta$

position vector for l_1

position vector for l_2

$$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta$$

$$\Rightarrow \quad 2+1+6 = \sqrt{14}\sqrt{6}\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{9}{\sqrt{14}\sqrt{6}} \quad \Rightarrow \theta = 11^\circ$$

8. A scientist is studying the effect of introducing a population of white-clawed crayfish into a population of single crayfish.

At time t years, the number of white clawed crayfish, w, and the number of signal crayfish, s, are modelled by the differential equations

$$\frac{dw}{dt} = \frac{5}{2}(w-s)$$

$$\frac{ds}{dt} = \frac{2}{5}w - 90e^{-t}$$

(a) Show that

$$2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t}$$ (3)

(b) Find a general solution for the number of white-clawed crayfish at time t years. (6)

(c) Find a general solution for the number of signal crayfish at time t years. (2)

The model predicts that, at time T years, the population of white-clawed crayfish will have died out.

Given that $w = 65$ and $s = 85$ when $t = 0$

(d) find the value of T, giving your answer to 3dp (6)

(e) Suggest a limitation of the model. (1)

Solution 8a

$$\frac{dw}{dt} = \frac{5}{2}w - \frac{5}{2}s \quad \text{and} \quad \frac{ds}{dt} = \frac{2}{5}w - 90e^{-t}$$

$$\Rightarrow \frac{d^2w}{dt^2} = \frac{5}{2}\frac{dw}{dt} - \frac{5}{2}\frac{ds}{dt}$$

$$= \frac{5}{2}\frac{dw}{dt} - \frac{5}{2}\left(\frac{2}{5}w - 90e^{-t}\right)$$

$$\Rightarrow 2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 5\left(\frac{2}{5}w - 90e^{-t}\right) = 0$$

$$\Rightarrow 2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w - 450e^{-t} = 0$$

$$\Rightarrow 2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t}$$

Solution 8b

$$\boxed{2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450\boxed{e^{-t}}} \quad (*)$$

$\Rightarrow$ Auxiliary equation is $\boxed{2m^2 - 5m + 2 = 0}$

$$\Rightarrow m = \frac{+5 \pm \sqrt{25 - 4\times2\times2}}{2\times2} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$

$$\Rightarrow m = 2, \tfrac{1}{2}$$

$$\Rightarrow \boxed{w = Ae^{0.5t} + Be^{2t}}$$

For particular integral (PI), try:

$$\boxed{w = ke^{-t}} \Rightarrow \frac{dw}{dt} = -ke^{-t} \Rightarrow \frac{d^2w}{dt^2} = ke^{-t}$$

$$(*) \Rightarrow 2(ke^{-t}) - 5(-ke^{-t}) + 2ke^{-t} = 450e^{-t}$$

$$\Rightarrow 9ke^{-t} = 450e^{-t} \Rightarrow k = 50 \Rightarrow \boxed{w = 50e^{-t}}$$

$$\Rightarrow \text{GS is } w = \boxed{Ae^{0.5t} + Be^{2t}} + \boxed{50e^{-t}}$$

Solution 8c

From part 8b, $\omega = \boxed{Ae^{2t}} + \boxed{Be^{0.5t}} + \boxed{50e^{-t}}$ (*)

$$\Rightarrow \frac{d\omega}{dt} = \boxed{2Ae^{2t}} + \boxed{\frac{B}{2}e^{0.5t}} \boxed{-\ 50e^{-t}} \quad (**)$$

From the question, $\omega = \frac{5}{2}(\omega - s)$

$$\Rightarrow \frac{2}{5}\frac{d\omega}{dt} = \omega - s$$

$$\Rightarrow s = \omega - \frac{2}{5}\frac{d\omega}{dt}$$

$$= \overbrace{Ae^{2t} + Be^{0.5t} + 50e^{-t}}^{(*)} - \frac{2}{5}\overbrace{\left(2Ae^{2t} + \frac{B}{2}e^{0.5t} - 50e^{-t}\right)}^{(**)}$$

tidying up

$$\Rightarrow s = \frac{1}{5}Ae^{2t} + \frac{4}{5}Be^{0.5t} + 70e^{-t} \quad (***)$$

Solution 8d

At $\omega = 65,\ s = 85,\ t = 0$:

$$(***) \Rightarrow s = \frac{1}{5}Ae^{0} + \frac{4}{5}Be^{0} + 70e^{0}$$

tidying up

$$\Rightarrow 85 = \frac{1}{5}A + \frac{4}{5}B + 70$$

$$\Rightarrow 15 = \frac{1}{5}A + \frac{4}{5}B$$

$$\Rightarrow 75 = A + 4B$$

$$\Rightarrow A = 75 - 4B \quad (****)$$

$$(*) \Rightarrow \omega = Ae^{0} + Be^{0} + 50e^{0}$$

$$\Rightarrow 65 = A + B + 50$$

$$\Rightarrow 15 = (75 - 4B) + B$$

$$\Rightarrow 15 = 75 - 3B$$

$$\Rightarrow 3B = 60 \Rightarrow B = 20$$

Substituting $B = 20$ in (****) $\Rightarrow A = -5$

Solution 8d continued

Plugging in values of A and B in (*) gives:

$$\boxed{\omega = -5e^{2t} + 20e^{0.5t} + 50e^{-t}}$$

At $\omega = 0$, $t = T$

$$\Rightarrow \boxed{0 = -5e^{2T} + 20e^{0.5T} + 50e^{-T}}$$

$\div(-5)$ $\Rightarrow 0 = e^{2T} - 4e^{0.5T} - 10e^{-T}$

$\times e^{-T}$ $\Rightarrow 0 = e^{3T} - 4e^{1.5T} - 10$

$$\Rightarrow 0 = (e^{1.5T})^2 - 4e^{1.5T} - 10$$

Let $u = e^{1.5T}$

$\Rightarrow 0 = u^2 - 4u - 10 \quad \Rightarrow u = 2 \pm \sqrt{14}$

$\Rightarrow e^{1.5T} = 2 \pm \sqrt{14}$

$\Rightarrow 1.5T = \ln(2 \pm \sqrt{14})$

take +ve root

$\Rightarrow T = \frac{1}{1.5}\ln(2+\sqrt{14}) \approx 1.165$